Math 250 Trig Review Practice – Day 2

1) Determine if the function is even, odd, or neither.

   a. $f(x) = \sin^2 x$

   b. $h(x) = \csc\left(x^2\right)$

   c. $g(x) = \sin x - \cos x$

2) Determine period, amplitude, all intercepts, vertical shift, and horizontal shift. Sketch graph.

   a. $y = -3\cos\left(x + \frac{\pi}{2}\right)$

   b. $y = 1 + \tan(2x)$

   c. $y = \csc(2\pi x)$

3) Verify the identity: $\sin^2 x - \sin^2 y = \cos^2 y - \cos^2 x$

M250 Trig Review – 2

①

even, odd, neither

a) $f(x) = \sin^2 x$

$f(-x) = \sin^2(-x)$

$= \sin(-x) \cdot \sin(-x)$

$= (-\sin x) \cdot (-\sin x)$ ← sign or odd/even identity

$= +\sin x \cdot \sin x$

$= \sin^2 x$

$= f(x)$

$f(-x) = f(x)$ so **even**

b) $h(x) = \csc(x^2)$

$h(-x) = \csc((-x)^2)$

$= \csc(x^2) = h(x)$

$h(-x) = h(x)$

even

whew! We dodged a bullet! But what would $\csc(-x)$ be?

$\csc(-x) = \frac{1}{\sin(-x)} = \frac{-1}{\sin x}$

$= -\csc(x)$

c) $g(x) = \sin x - \cos x$

$g(-x) = \sin(-x) - \cos(-x)$

$= -\sin x - \cos x$ ← sign or odd/even identity

$= -(\sin x + \cos x)$

$\neq g(x), \neq -g(x)$ **neither**

Math 250 Trig Review – Day 2

(2) Determine period, amplitude, all intercepts, vertical shift and horizontal shift. Sketch graph.

a) $y = -3\cos\left(x + \frac{\pi}{2}\right)$

period = 2π (argument x has coefficient 1)

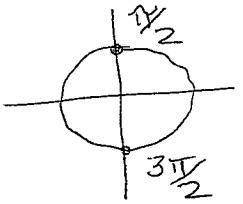
amplitude = 3 (coefficient of cosine, absolute value)

x-intercepts: set $y = 0$

$0 = -3\cos\left(x + \frac{\pi}{2}\right)$ (divide by -3)

$0 = \cos\left(x + \frac{\pi}{2}\right)$

angle



cosine is 0 when angle is $\frac{\pi}{2}$ or $\frac{3\pi}{2}$

$x + \frac{\pi}{2} = \frac{\pi}{2}$ $\qquad$ $x + \frac{\pi}{2} = \frac{3\pi}{2}$

$x = 0$ $\qquad$ $x = \pi$

graph crosses x-axis with period 2π

$0, 2\pi, 4\pi, 6\pi, \ldots$
$\pi, 3\pi, 5\pi, \ldots$
} together, this is $0, \pi, 2\pi, 3\pi \ldots$

x-intercepts $(k\pi, 0)\ k \in \mathbb{Z}$

y-intercepts: set $x = 0$

$y = -3\cos\left(0 + \frac{\pi}{2}\right)$

$= -3\cos\left(\frac{\pi}{2}\right)$

$= -3 \cdot 0$

$= 0$

y-intercept $(0,0)$

Vertical shift: none (nothing added to trig function)

horizontal shift: $x + \frac{\pi}{2} = 0$ set argument = 0

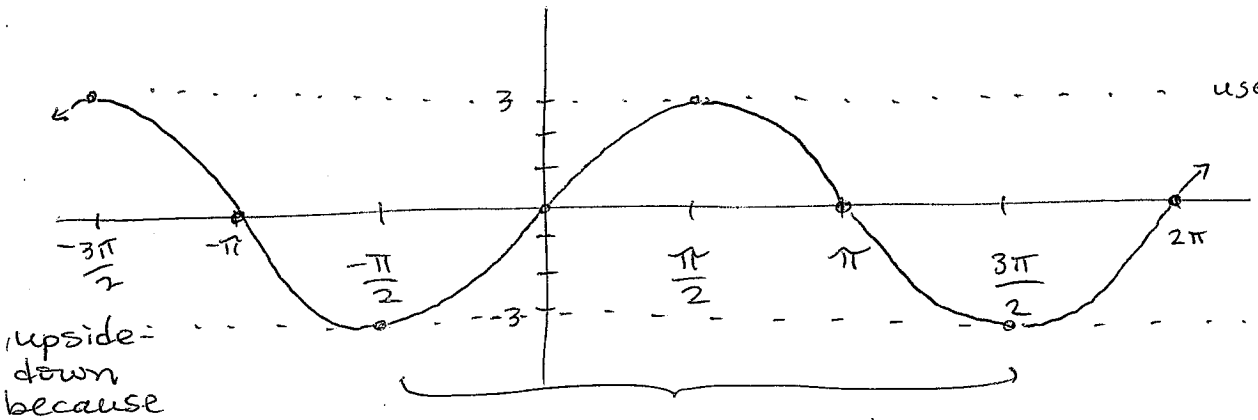
$x = -\frac{\pi}{2}$ solve for x

horizontal shift = $-\frac{\pi}{2}$ or $\frac{\pi}{2}$ to left

Math 250 Trig Review – Day 2

② a) cont

Graph.



upside-down because $-3 < 0$ neg

one period $= 2\pi$
divided into 4 equal segments

② b) $y = 1 + \tan(2x)$

period: $2x = \pi$

$x = \boxed{\frac{\pi}{2} \text{ period}}$

(set argument = original period of tanx and solve for variable)

amplitude: none (tan, cot, sec, csc never have amplitude)

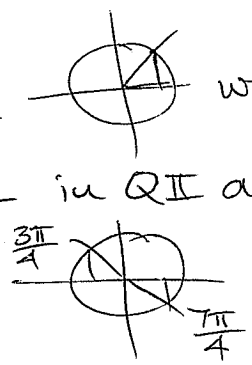
x-intercepts: set $y = 0$

$0 = 1 + \tan(2x)$

$-1 = \tan(\underbrace{2x}_{\text{angle}})$

$\tan(\theta) = 1$ when $\sin\theta = \cos\theta$

$\tan(\theta) = -1$ in QII and QIV



when angle is $\frac{3\pi}{4}$ or $\frac{7\pi}{4}$

$2x = \frac{3\pi}{4}$ $\quad 2x = \frac{7\pi}{4}$

$x = \frac{3\pi}{8}$ $\quad x = \frac{7\pi}{8}$

graph crosses x-axis with period $\frac{\pi}{2}$

$\frac{3\pi}{8} + \frac{\pi}{2} = \frac{7\pi}{8}$ ... $\frac{3\pi}{8}, \frac{7\pi}{8}, \frac{11\pi}{8}, \frac{15\pi}{8}, \ldots$

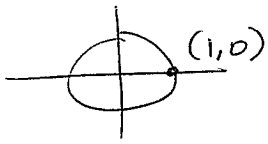
x-intercepts $\left(\frac{3\pi}{8} + \frac{k\pi}{2}, 0\right)$ $k \in \mathbb{Z}$

y-intercepts: set $x = 0$

$y = 1 + \tan(2 \cdot 0)$

Math 250 Trig Review - Day 2

(2) b) continued.



$\tan(0) = \frac{0}{1} = 0$

$y = 1 + \tan(0)$

$y = 1 + 0$

$y = 1$

y intercept (0,1)

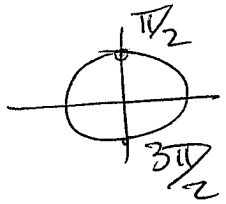
vertical shift : up 1 ($\tan(2x)$ has been added to 1)

horizontal shift : none (argument 2x has no addition or subtraction)

Graph.

Need asymptotes. $\tan(2x)$ undefined where $\frac{\sin(2x)}{\cos(2x)}$ undefined

where $\cos(\underbrace{2x}_{\text{angle}}) = 0$



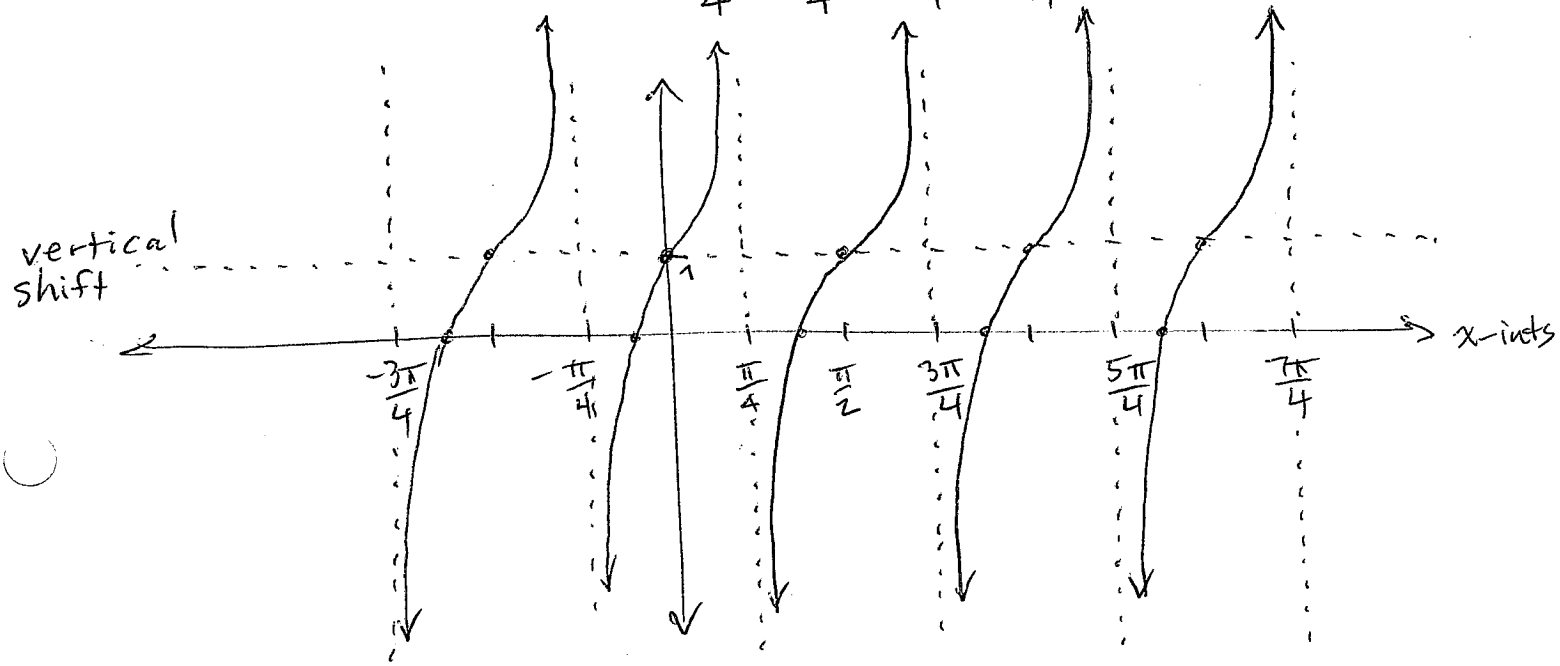
where angle

$2x = \frac{\pi}{2}$ $\quad 2x = \frac{3\pi}{2}$

$x = \frac{\pi}{4}$ $\quad x = \frac{3\pi}{4}$

which recur with period $\frac{\pi}{2}$ ....

$\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}, \ldots$



c) $y = \csc(2\pi x)$

period $\quad 2\pi x = 2\pi$

$x = 1$ $\quad$ period 1

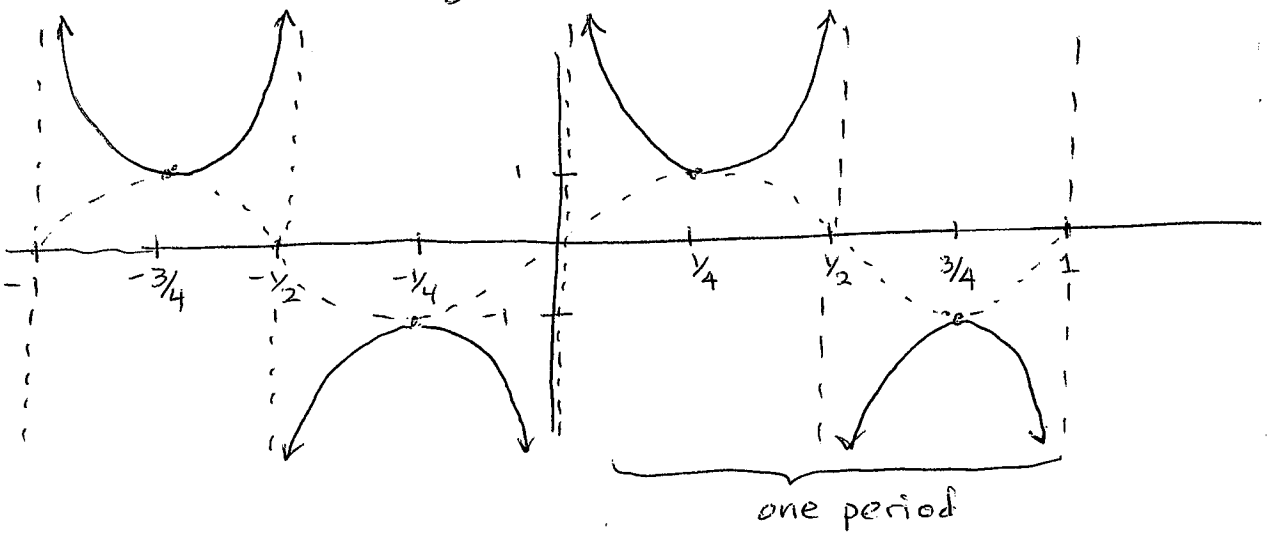
amplitude: none

vertical shift: none

horizontal shift: none

intercepts: no x, no y

graph: helping function $\sin(2\pi x)$



(3) Verify $\sin^2 x - \sin^2 y = \cos^2 y - \cos^2 x$

Beginning with LHS, substitute known identities until the result is the RHS.

$\sin^2 x - \sin^2 y$ ← we have $\sin^2$, want $\cos^2$

use $\sin^2 x + \cos^2 x = 1$

algebra: $\sin^2 x = 1 - \cos^2 x$

ditto $\sin^2 y = 1 - \cos^2 y$

$= (1 - \cos^2 x) - (1 - \cos^2 y)$

$= 1 - \cos^2 x - 1 + \cos^2 y$

$= -\cos^2 x + \cos^2 y$

$= \cos^2 y - \cos^2 x$ ✓